COGNITIVE PSYCHOLOGY 1, 1–17 (1970)

Second-Order Isomorphism of Internal Representations: Shapes of States[1]

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It is argued that, while there is no structural resemblance between an individual internal representation and its corresponding external object, an approximate parallelism should nevertheless hold between the relations among different internal representations and the relations among their corresponding external objects. In support of this "second-order" type of isomorphism, subjective judgments of the similarities among the shapes of 15 states of the U. S. are found (*a*) to be very much the same whether the states to be compared are pictorially displayed or only imagined, and (*b*) to be related, in both cases to identifiable properties of their actual cartographic shapes.

A Discredited "First-Order" Concept of Isomorphism

We are apt to say of a square—not only it *is* square—but also that we *see* or *remember* it as square. It is tempting to suppose that we must at such times have some sort of internal representation or mental image that is itself square. However, the grounds underlying such an argument for this concrete kind of structural isomorphism was effectively cut away by Skinner (1945, 1963), Wittgenstein (1953), and others who have pointed out that we learn the appropriate use of such words as "square" from a verbal community that has access only to the public object and not to any such private image.

[1] The work reported here was supported by National Science Foundation Grants GS-2283 to Stanford University and GS-1302 to Harvard University, where the first author was principal investigator at the time this work was initiated. The first author is responsible for the conception of this work and its written report; the second author carried out the experiment and most of the analyses of the data. We are indebted to Jih-Jie Chang and J. Douglas Carroll for programming an adaptation of Cliff's method for rotation to congruence, and to Allen Newell for helpful suggestions concerning the manuscript.

Indeed, if there is, as we suppose, some internal event that corresponds to our perception of a square, our ability to form an association between this event and the word "square" requires only that this event have a regular relation to the external object is one of causality, not of structural isomorphism. Such an event *could* be the activation of some group of neurons, such, perhaps, as a "cell assembly" of the sort described by Hebb (1949). To insist, in addition, that these neurons must be spatially arranged in precisely the form of a square, themselves, does not in the least help to explain how they come to trigger the naming response "square." On the contrary, it only attempts the absurdity of putting off until later the whole process of pattern recognition that must by definition preceed the pivotal event in question. (With about as much logic, one might as well argue that the neurons that signal that the square is green should themselves be green!)

A Proposed "Second-Order" Concept of Isomorphism

One of us has recently proposed that the general concept of isomorphism can still be saved from the kinds of objections just raised (e.g., see Shepard, 1968, pp. 287–288). The crucial step consists in accepting that the isomorphism should be sought—not in the first-order relation between (*a*) an individual object, and (*b*) its corresponding internal representation—but in the second-order relation between (*a*) the relations among alternative external objects, and (*b*) the relations among their corresponding internal representations. Thus, although the internal representation for a square need not itself be square, it should (whatever it is) at least have a closer functional relation to the internal representation for a rectangle than to that, say, for a green flash or the taste of persimmon.

By the "functional relation" between two internal representations we refer, most fundamentally, to the tendency of a response that has been directly associated with one to be aroused, also, by the activation of the other. However, it is a fact of inadequately appreciated significance that, despite the practically unlimited range and diversity of possible internal representations, we can readily assess within ourselves the degree of functional relation between any two by a simple, direct judgment of subjective similarity. Moreover, we can do this even though (*a*) we have never before compared the two representations in question, and even though (*b*) we may be unable to communicate anything about the absolute nature of either of the two representations taken separately. (Thus, we easily report that orange is more similar to red than to blue without being able to say anything significant either about the unique subjective experience of the color orange itself or, of course, about the unique pattern of neural activity that presumably underlies this experience.) One could even turn the matter around and argue that it is primitive, internal assessments of similarity of this sort (implicit or even unconscious though they may often be) that mediate every response we make to any situation that is not exactly identical to one confronted before.

In any case, for the relations among internal representations to mirror, in this functional way, at least the biologically more significant relations among the corresponding external stimuli would clearly be of adaptive utility. And this conclusion is not vitiated by the fact that the isomorphism is not perfect; i.e., by the fact that some properties of the physical world are reflected only weakly, if at all, within the internal system. The same conclusion applies to internal representations of objects or events that are not physically present, but only remembered or imagined—at least if we suppose that these, too, play some role in governing our overt behavior.

The following experiment is intended to illustrate a technique whereby it should be possible to learn something about the functional relations among internal representations retrieved from long-term memory and, moreover, to assess the extent to which these relations are (in the proposed, abstract sense) isomorphic both (*a*) to those among the corresponding representations of direct perception, and even (*b*) to certain discernible relations among the corresponding external objects.

METHOD

Most educated citizens of the United States have at least some notion of the shapes of many of the 50 states. Indeed, without even looking at a map, they readily "see" that Oregon is more similar in shape to Colorado than to Florida. The use of the word "see" seems not inappropriate here in that *S*s often report that such judgments are based upon direct comparisons between mental images (however imperfect or vague) of the shapes in question. In fact, although it is also to some extent possible to describe some of these shapes in words, this seems to require an additional effort that (*a*) often is not even attempted unless explicitly required, and (*b*) seems, when it *is* attempted, to be guided, at least in part, by reference to the appropriate visual images, which typically have come to mind first. In the present experiment, judgments of similarity in shape were collected for all pairs of 15 states under two conditions: one in which the shapes were pictorially displayed for direct visual comparison, and another in which they were available, if at all, only in imagination.

Subjects

Seven graduate students at Harvard University served as *S*s. They were all personally known to at least one of the experimenters, had received their education in the United States, and were believed to be both somewhat familiar with the shapes of the states used in the experiment and, also, sufficiently motivated to carry through the rather demanding sorting task required.

Stimuli

The 15 states with respect to which judgments of similarity were sought were Alabama, Colorado, Florida, Idaho, Illinois, Louisiana, Maine, Minnesota, Missouri, Nebraska, Nevada, Oklahoma, Oregon, South Carolina, and West Virginia. These were selected, in part, because they did not vary too greatly in size and yet seemed representative of a wide variety of shapes. Outline drawings of each of these 15 states were traced from a map of the United States and were then duplicated so that they could be glued on 3 × 5-in cards in pairs. Two decks of 105 cards each were prepared—the "picture" deck and the "name" deck. In the picture deck, each of the 105 possible pairs of different states appeared on a different one of the 105 cards, side-by-side, merely as outline drawings (without any identifying names), and in conventional orientation (with north at the top). In the name deck, each of the 105 possible pairs of these same states were also represented on a different card of the deck, but this time in the form of the side-by-side printed names of those two states only.

Procedure

For each *S*, the deck selected on a given occasion would first be shuffled and then handed to the *S* with the instruction to rank order the 105 cards according to the similarity of the shapes of the two states represented (whether by name or picture) on each card so that, when finished, the two states judged to be most similar in shape would be those represented on the top card, the two judged to be the next most similar in shape would be those represented on the second card, and so on for all 105 cards. (The *S*s complained that this was a trying task, which is not surprising since they were attempting to come at least close to the best out of a total set of 105 factorial possible rank orders.)

All *S*s were given the name deck first and asked to complete their sorting of that deck before beginning on the picture deck. Thus the internal representations or mental images, if any, with respect to which the *S*s made their judgments under the name condition were necessarily drawn from long-term rather than short-term memory. The entire procedure of sorting the two decks took from 45 min to a little over an hour, depending on the individual *S*.

RESULTS

The two principal questions to be answered concern, first, the extent to which the judgments of similarity in shape were alike whether the shapes to be compared were actually presented or only named and, second, the extent to which both sets of judgments reflect identifiable properties of the actual shapes. These two questions are taken up successively in the two following sections.

Relations between the Similarity Data from the Two Conditions

For purposes of overall comparison between the name condition and the picture condition, an average rank number was computed (over the seven *S*s) for each of the 105 pairs of states under each of the two conditions. Theoretically, these average rank numbers could have ranged from 1 (for the two states judged most similar in shape) to 105 (for the two judged least similar) under each condition. However, since different *S*s produced somewhat different rankings, the ranges of the resulting numbers were somewhat compressed and, in fact, extended only from 14 to 93 for the name condition, and from 2 to 93 for the picture condition.

Despite this compression, the two sets of average rank numbers were clearly related to each other. Thus Colorado and Oregon, which ranked first in similarity according to their average rank number for the name condition (14), also ranked first for the picture condition (2); Nebraska and Oklahoma, which ranked second for the name condition (18), at least attained a tie for second place for the picture condition (9); and, at the other extreme, Oregon and West Virginia, which ranked last for the name condition (93), came only sixth from last place out of the 105 average rank numbers for the picture condition (82). Over all, the product-moment correlation between the 105 average rank numbers for the name condition and the 105 average rank numbers for the picture condition was +.73.

This correlation does of course attain a high level of statistical significance. However, the more interesting question remains as to whether the failure of this correlation to approach 1.0 is attributable merely to the essentially random fluctuations to be expected in any judgmental data, or whether it reflects an appreciable systematic difference between the judgments made under the two conditions. In order to obtain evidence bearing on this question, a number of more detailed analyses were carried out.

First, product-moment correlations were computed among all 2×7 individual sets of rank numbers (produced by each of the seven *S*s under each of the two conditions), as well as between each of these 14 individual sets and each of the two sets of average rank numbers just considered. (These computed correlations may tend to underestimate the true underlying correlations since the rank numbers are equally spaced and, so, do not reflect the fact that discrimination of relative similarity was generally better for the most similar pairs. However, this would not be expected to affect the merely ordinal comparisons among conditions presented below.)

Second, each of the 14 sets of individual rank numbers as well as the two sets of average rank numbers were treated as ordinal "proximity" data and separately subjected to multidimensional scaling of the Shepard-Kruskal variety (Kruskal, 1964; Shepard, 1962) using, in particular, Kruskal's "M-D-SCAL" program. In this section we are concerned only with the two-dimensional results which, as we claim in the next section, provided generally adequate representations of the data. More particularly, we are here concerned only with the numerical value of the residual departure of each set of data from a perfect monotonic fit to its two-dimensional solution. For, to the extent that this low-dimensional model is appropriate, this value—called the "stress" of the solution (Kruskal, 1964)—provides an indication of the degree of internal inconsistency of the particular set of judgments. (To take an extreme example, if the pairs A-B and B-C are ranked as the two most similar of all 105 pairs, the pair A-C cannot then be ranked as the very least similar without violating the strong internal constraints of the low-dimensional model and, hence, contributing to the resulting "stress.")

The results of these further analyses are summarized, in part, in Table 1. Columns A and B present, for the name and picture conditions, respectively, one minus the two-dimensional stress for each *S* and (in the very bottom row) for the two solutions based upon the average data for the group as a whole. The stress values are subtracted from one to yield a measure of consistency (rather than inconsistency) that is more directly comparable to the correlations presented elsewhere in the table.

Columns C and D present the product-moment correlations between each set of individual data and the corresponding average data for the whole group under the name and picture conditions, respectively. As indicated to the left of the table, the entire rows corresponding to the seven individual *S*s have been permuted so that the *S*s are arranged from "best" (at the top) to "worst" (at the bottom) solely on the basis of the average of these two measures of the extent to which each *S* is representative of the whole group.

The remaining columns present the correlations between the sets of rank numbers for the two *different* conditions, depending upon whether both sets of data were from the individual *S* (Column G), or whether the

TABLE 1

Measures of Consistency Within the Name and Picture Conditions Separately, and Measures of Correlation Between These Two Conditions (presented for individual subjects and for the group of seven as a whole)

Subjects	A	B	C	D	E	F	G
	Consistency with self (1.0—stress in two dimensions)		Consistency with group (correlation with group average)		Correlations between conditions		
Ordered according to consistency with group (Cols. C & D)	Name condition	Picture condition	Name condition	Picture condition	Individual name vs. group picture	Individual picture vs. group name	Individual name vs. individual picture
4	.86	.87	.79	.86	.76	.79	.85
3	.78	.75	.77	.82	.73	.68	.73
2	.81	.76	.73	.76	.57	.61	.65
1	.80	.80	.72	.71	.58	.49	.36
5	.84	.79	.51	.67	.33	.41	.22
6	.75	.79	.45	.50	.22	.24	.17
7	.70	.72	−.10	.55	−.36	.35	−.31
Median	.80	.79	.72	.71	.57	.49	.36
Mean (all *S*s)	.79	.78	.55	.70	.40	.51	.38
Mean (top 3)	.82	.79	.76	.81	.69	.69	.74
Whole group	.86	.84	(1.00)	(1.00)	Group name vs. Group picture	=	.73

name or picture data were the averages for the whole group (Columns F and E, respectively).

It is apparent, particularly from the correlations (Columns C through G), that the *S*s differed quite consistently in the accuracies of their judgments. Thus, among all seven *S*s, *S* 4 was uniformly highest with respect to internal consistency (Columns A and B), "external" consistency (C and D), and agreement between the two conditions (E, F, and G); while *S*s 6 and 7 were uniformly lowest in all these respects—with the single exception of the value for *S* 6 in Column B. (Indeed, out of the total set of 120 separate correlations summarized in Tables 1 and 2, the only ones with negative sign were those contributed by *S* 7 under the name condition.)

The fact that Column D generally agrees with the other columns suggests that the marked individual differences cannot be primarily attributed to differences in ability to imagine the shapes of the states. For, Column D pertains only to the condition in which the shapes were actually presented and, so, did not have to be imagined. The differences could better be explained as a reflection of some more pervasive factor such, perhaps, as care taken in sorting the pairs on the basis of similarity—under whatever condition. Still, it is undoubtedly the case that the *S*s did differ in the accuracy of their knowledge of the shapes of the states, and future studies might profitably include a test of each *S*'s ability to identify the objects (e.g., states) as a way of gauging this source of variation.

TABLE 2
Measures of Central Tendency and Dispersion of the Distribution of All Correlations Between Individual Rankings for Each Possible Combination of Conditions (name-name, picture-picture, and name-picture)

Combination of conditions correlated	A	B	C	D	E	F	G	H
	For all seven *S*s				For top three *S*s			
	Median	Mean	SD	*N*	Median	Mean	SD	*N*
1 One *S*'s name vs. other *S*'s name	.16	.19	.34	21	.65	.60	.11	3
2 One *S*'s picture vs. other *S*'s picture	.40	.40	.18	21	.66	.62	.10	3
3 1 and 2, above, combined	.32	.29	.29	42	.65	.61	.10	6
4 One *S*'s name vs. other *S*'s picture	.30	.26	.27	42	.63	.60	.10	6
5 One *S*'s name vs. same *S*'s picture	.36	.38	.40	7	.73	.74	.10	3

Whatever the source of the individual differences may be, here their consistency provides some justification for a cautious look at the results for a subset containing just the most accurate *S*s. Accordingly, data for the three *S*s who were most representative of the whole group, as indicated by Columns C and D, are displayed separately in the next-to-last row of Table 1 and, also, in the right-hand half of Table 2.

With regard to the principal question under consideration, it is suggestive that many of the correlations in Columns E and F of Table 1 (which relate the data for each *S* under one condition to the data for the whole group under the other condition) are not very much lower than the correlations in Columns C and D (which relate the data for each *S* under one condition to the data for the whole group under the *same* condition). For, it must be noted that the latter correlations (but not the former) are inflated by the direct contribution of each individual's data to those for the whole group.

A more decisive comparison, free of this kind of statistical contamination, is afforded by the summary statistics in Table 2. Columns A and B in that Table present the median and mean of all correlations between two individual sets of rank numbers of the sort specified at the left of the table; and Columns E and F do the same for the three *S*s who were most representative of the whole group according to Table 1. Columns C and G present the standard deviation of the distribution of all appropriate correlations belonging to each specified combination; and Columns D and H display simply the number of appropriate correlations belonging to each combination (upon which the statistics in the other columns were based).

On the average (Columns A and B), it is clear that there was generally less agreement between different *S*s under the name condition (Row 1) than under the picture condition (Row 2). However, this is less true for the three "best" *S*s (Columns E and F). Moreover, such a difference could merely reflect the likely fact that judgments were generally more variable under the condition in which the shapes to be judged were not actually presented. Certainly it does not imply that the *S*s were doing qualitatively different things under the two conditions. On the other hand, the fact that each *S* tended to agree more with himself (Row 5) than with a different *S* (Row 4) does suggest that different *S*s may have been doing slightly different things.

Most crucial of all is the comparison between Rows 3 and 4. Here we see that the average correlation between the judgments of similarity made by two different *S*s under the same condition (Row 3) is no more than about one-tenth of a standard deviation above that for two different *S*s under different conditions (Row 4). Moreover, this is true whether we average over all 42 of the appropriate correlations for the whole group (Columns A and B) or only over the six of these correlations pertaining to the three most representative *S*'s (Columns E and F). This comparison does seem, then, to weigh heavily against the likelihood of any substantial systematic differences in the judgments made under the name and picture conditions. Still further evidence against such a likelihood will be brought out, incidentally, in the multidimensional scaling results described in the following section.

Relations of the Similarity Data to the External Shapes

Although the preceding analyses support the hypothesis that the *S*s were making similar kinds of comparisons under the two conditions, they do not provide any evidence bearing on the further hypothesis that these comparisons were among internal representations that could be said, in the proposed abstract sense, to be isomorphic to the corresponding external shapes. The same degree of agreement could have resulted if, contrary to both instruction and introspection, the *S*s were basing their judgments under both conditions upon something that has no intrinsic connection with the shapes—such as the similarities of the arbitrarily associated names of the states (cf. Conrad, 1964), or associations between the states deriving from some extraneous factor such as geographical location.

An advantage of the multidimensional scaling analyses is that they do provide a convenient way of pursuing this question of isomorphism. For this purpose we shall be primarily concerned with the analyses of the two sets of average rank numbers (for the group as a whole, under the name and picture conditions).

For each of these two sets of data, best-fitting solutions were obtained in 1, 2, 3, 4, and 5 dimensions separately. When listed in this order, the stress values for the solutions were .35, .14, .10, .06, and .06 for the name condition, and .37, .16, .09, .06, and .05 for the picture condition. Thus, for each specified number of dimensions, the fits achieved were of about the same quality for the name and for the picture conditions. In both cases, the fit in one dimension seems unsatisfactory while the fit in two dimensions is acceptable and, anyway, does not improve in any abrupt way as we proceed on to three or more dimensions. Moreover, the three-dimensional solutions did not seem to provide any clearer insight into the data than the two-dimensional solutions. And finally, as in the case of other methods of reducing or averaging data, a solution with fewer degrees of freedom generally tends to provide a more stable (if slightly distorted) estimate of the true underlying structure. Accordingly, we continue to confine ourselves here to a consideration of the two-dimensional results.

The results of such multidimensional scaling are determined only up to an arbitrary rigid motion and overall change of scale. Consequently, before attempting a comparison between the two obtained 15-point configurations, it is first necessary to bring them into a good mutual fit by means of an appropriate similarity transformation (including a rigid rotation as well, possibly, as a translation, reflection, and uniform expansion or contraction). In the present case this was carried out objectively by means of an adaptation of Cliff's (1966) least squares method for "orthogonal rotation to congruence." Specifically, the two-dimensional solution obtained for the name condition was taken as fixed, "target" configuration while the solution obtained for the picture condition was transformed into a best fit with that target. (Essentially identical results are obtained if the name configuration is fitted to the picture configuration.)

The final configurations are displayed, as thus superimposed, in Fig. 1. Each of the 15 states is represented by an arrow positioned in such a way that the tail end, which is surrounded by the actual outlines of the state, coincides with the point recovered from the picture condition; and the head end, which is accompanied by the abbreviated name of that state, coincides with the point recovered from the name condition. The arrows thus provide an indication of how much the relative positions of the shapes implied by the judgments of similarity can be expected to move away, from what is obtained under the condition in which those shapes are actually presented, when they are only named.

The positions of some of the states did move to a noticeable extent (especially, in the present sample, that for Nevada). Over all, however, the relations among the points representing the states remained remarkably the same. As a quantitative coefficient of agreement between the two configurations we can take $\Sigma xy^*/\Sigma x^2$, where x denotes the normalized coordinates of the target configuration (obtained from the name condition) and where y^* denotes the normalized coordinates of the transformed configuration (obtained from the picture condition). In the present case this coefficient is .917. That it is this close to unity provides a somewhat different kind of support for the conclusions already reached; namely, that there was no appreciable systematic difference between the two sets of similarity data.

In addition, though, the results presented in Fig. 1 provide strong evidence for the hypothesis that the judgments were, under both conditions, based upon geometrical properties of the actual cartographic shapes. Note in particular, that all smallish, irregularly shaped states with wiggly

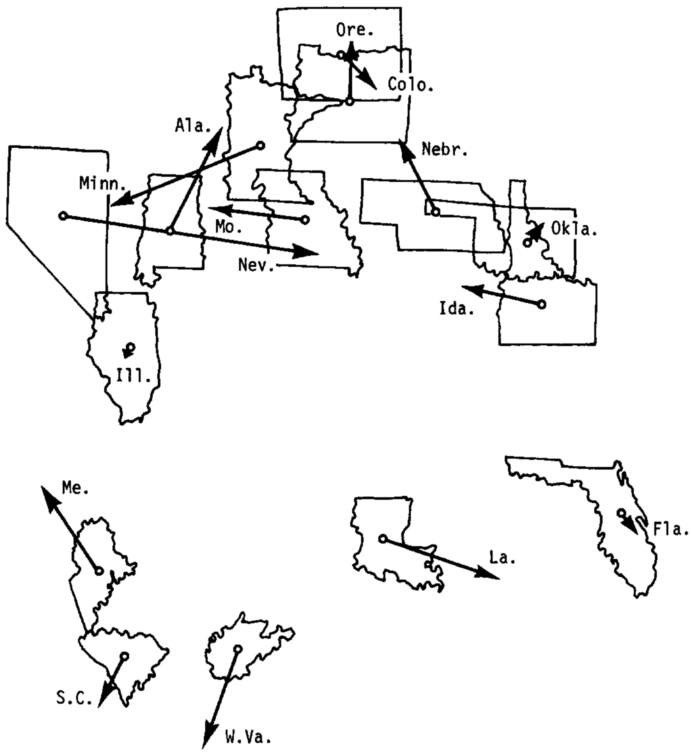

FIG. 1. Superimposed two-dimensional solutions for the name condition (heads of arrows) and the picture condition (tails of arrows) based upon the two sets of similarity data averaged over all seven *S*s.

borders are spread across the bottom (Me., S. C., W. Va., La., and Fla.); that all roughly rectangular, horizontally elongated states with mostly straightish borders are grouped at the top (Colo., Ore., Neb., and Okla.); that more or less irregularly shaped states with predominantly vertical elongation appear together on the left (Ala., Ill., Minn., Mo., and Nev.); and that all states with a pronounced "handle," "elbow," or "L-shape" are located on the right (Neb., Okla., Ida., Fla., and La.). Moreover, within these four major groupings, states are to some extent arranged in a manner consistent with the neighboring groups. (Thus, within this last group of "L-shaped" states, those that are larger and more rectangular appear with the other largish rectangular states at the top, while those that are smaller and more irregular appear with the other smallish irregular states at the bottom).

As might be expected from the positive correlations in Columns C and D of Table 1, the two-dimensional solutions for at least the better of the individual *S*s were in generally good agreement with the group solutions displayed in Fig. 1. Figure 2 shows the two solutions, superimposed as in Fig. 1, based upon the group data for just the three best *S*s. The coefficient of agreement between the name and picture configurations for these *S*s is even better, .941, and the same general pattern and groupings emerges much as in Fig. 1.

We also examined the two-dimensional configurations obtained for the most deviant *S* (No. 7) in order to discover whether this *S* was making his judgments on some entirely different basis. However, the only pattern that we could discern was that certain states seemed to group together as for the other *S*'s (e.g., into the groups Ill., Minn., and Mo.; La., Fla., and S. C.; Me. and W. Va.; or Okla. and Nebr.). Beyond this, the pattern seemed more or less random (and as unrelated to the names or geographical location of the states as to their shapes). Again, then, it may be that the judgments of the least consistent *S*s were simply more erratic—owing, perhaps, to lack of care in making the judgments compounded, possibly, by relatively poorly defined mental images.

The 15 shapes used here constituted a very special set and, so, it is

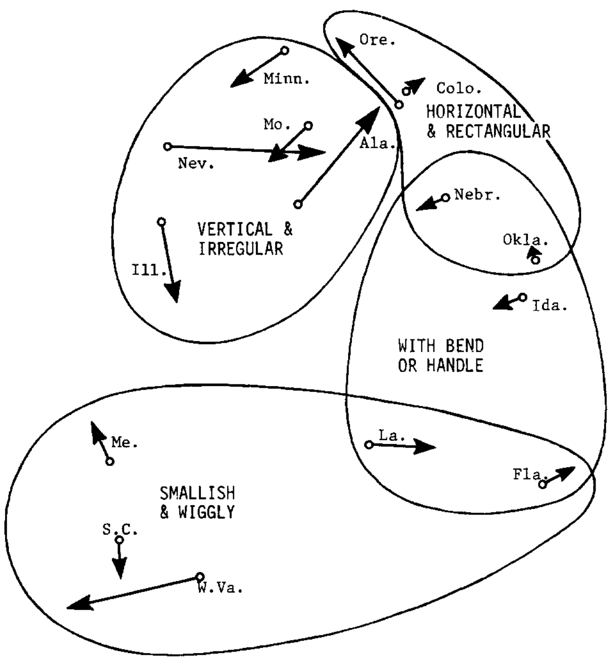

FIG. 2. Superimposed two-dimensional solutions (indicated by arrows as in Fig. 1) based upon the two sets of similarity data averaged over the three "best" *S*s only.

not necessarily to be expected that the properties or dimensions of the shapes that seemed to govern the *S*s' judgments would be the same as those found to be important for other sets of shapes. Nevertheless, there does appear to be some connection between most of the properties noted above and the three dimensions that Behrman and Brown (1968) have recently put forward as accounting for judgments of similarity for a representative sample of four-sided polygons. Certainly, the factor of vertical or horizontal elongation seems directly related to their dimension of "maximum ratio of vertical to horizontal extent"; and the present additional factors of irregularity or "wiggleness" of border and presence of an "elbow" (or reentrant angle) both seem related to their other two dimensions of "dispersion away from the centroid" and "jaggedness" (i.e., variance of interior angles).

Indeed, before we had even seen the report by Behrman and Brown, we had tentatively interpreted three orthogonal directions in our best-fitting three-dimensional solutions (not presented here) as dimensions of "vertical versus horizontal elongation," "general irregularity of form versus rectangularity," and "wiggliness versus straightness of borders." These appear, even more clearly, to correspond in a rough way to their three dimensions of "elongation," "dispersion," and "jaggedness."

In any case, it is clear from the results of multidimensional scaling shown here in Fig. 1 that, even under the condition in which only the names of the states were presented, the states that were judged to be most similar were those (such as Oregon and Colorado) that have similar shapes and not those (such as Alabama and Louisiana) that have somewhat similar sounding names or similar associations.

DISCUSSION AND CONCLUSIONS

The above results have implications of essentially the logical form, though nearly converse content, of those of Conrad (1964). He demonstrated that confusions in the recall of certain visually presented shapes (viz., letters) were more strongly related to acoustic features of the names of those shapes than to visual features of the shapes themselves. And this was taken as evidence that the internal representations were, in the present sense, more isomorphic to the implicit names of those shapes than to the explicitly presented shapes. We, on the other hand, have demonstrated that, when only the names of certain shapes are presented, judgments of similarity can be more strongly determined by visual features of the shapes named than by any features of the names themselves. And this is taken as indicating that the internal representations studies here were more isomorphic to the unpresented shapes than to explicitly presented names. The two cases are alike, however, in providing evidence that overt behavior is often guided by internal representations whose interrelations mirror relations among objects in the external world other than those actually presented to the *S*s.

In the present case, the results are consistent with introspective indications that these internal representations are what we normally refer to as visual images. However, in view of the likely fact that the *S*s could recognize at least some of the shapes tactually as well as visually, such "images" are probably not as modality specific as the term "visual" may suggest above. Whatever their precise nature, we can say of these internal representations or images that, while they were generated directly via perception under the picture condition, they were regenerated only indirectly via long-term memory under the name condition.

Possibilities for Meeting Certain Objections

We would not, however, deny that considerable further work will be needed in order to rule out the possibility that, contrary to introspective indications, the judgments in the "name" condition (or perhaps even in both conditions alike) were based on a comparison—not of mental images in the usual sense—but of implicit verbal descriptions. One way to evaluate this possibility would be to conduct a similar experiment with stimuli that are even less amenable to verbal encoding. Sounds of orchestral instruments, photographs of familiar faces, or distinctive odors, for example, while easy to name or to identify, might prove sufficiently difficult to capture by purely verbal description.

Since colors, also, tend to be relatively "unanalyzable" in this sense (Shepard, 1964, pp. 80–81), the results of a recent unpublished study by Rapoport and Fillenbaum (1968) may be relevant here. Although their report came to our attention only after the completion of the experiment reported here, it describes a remarkably similar experiment using the names of 15 different colors. In effect, *S*s were required to rank-order all 105 pairs of these 15 color words on the basis of the subjective similarities between the two colors named by each pair of words. Despite the relatively less "analyzable" character of the referents of their words and in accordance with the kind of "isomorphism" advocated here, they obtained a two-dimensional solution for their similarity data that strongly resembled "color circles" previously obtained (e.g., by Shepard, 1962, p. 236) by analyzing judgments of similarity from experiments in which the colors were actually presented.

Another question that could be raised concerns whether the *S*s in the present experiment might not have modified their judgments under the picture condition just so that these judgments would agree more closely with the judgments they had already made under the preceding name condition. However, it seems a bit implausible, even if the states were all recognized from their shapes, that the *S*s could remember much about how they had previously ranked each of the 105 pairs of names. Still, future studies probably should take the precaution of sometimes presenting the picture condition either alone or *before* the name condition. It might even be possible to embed the pairs of pictures in a series of pairs of similar nonsense figures so that they are never identified for what they are (e.g., states). This would also rule out the further possibility that the judgments were to some extent contaminated by extraneous associations (e.g., concerning whether the states begin with the same letter, are located in the same geographical region, or are known for the same kind of product, economy, scenery, etc.). In the present case, although these two uncontrolled sources of influence might be regarded as contributing something to the agreement between the two sets of similarity data, they do not account at all for the evident relation that both sets of data bear to the actual geometrical properties of the shapes judged.

Finally, since some of the *S*s did complain that their mental images of some of the 15 states were not very sharply defined, it would be desirable to find stimuli with which the *S*s are more thoroughly familiar. Again, for some *S*s at least, sounds of orchestral instruments or well-known faces might meet this requirement. In any case, as we already noted, an additional test of each *S*s' ability to identify the stimuli could provide further useful information.

Concluding Remarks

At the philosophical level, our position is consonant with the neural-identity theory of the mental events as set forth by U. T. Place (1956) and J. J. C. Smart (1959). According to this position, when someone says "I am imagining a green square," he is not telling us that there is anything that is literally either green or square going on in his head. What he is really telling us is that whatever is going on is functionally similar to what usually goes on when he is confronted with the kind of external object to which we have all associated the words "green" and "square."

To be still more precise, these words have been directly attached—not to the external object—but to the internal (perceptual) events to which the external object usually leads. From there, according to the present view, the words generalize to the functionally related internal events that we call the "image" of a green square that is not physically present but only remembered or imagined.

Now the reason that purely mental images of this latter sort have seemed so inaccessible to empirical study may have stemmed from a hastily formed notion that such a study should concern the nature of a mental image taken by itself. And, *S*s do indeed seem unable to tell us anything significant about the structure of an individual mental image as such. What they can, however, tell us about is the relations between that internal representation and other internal representations. In this sense, then, we would extend to memory and imagination what Garner (1966, p. 11) has already asserted with regard to direct perception; namely, that "the factors known in perception are properties of sets of stimuli, not properties of individual stimuli." We hope that the present study at least demonstrates the possibility of investigating the structure of internal representations at this more abstract level and, at the same time, provides evidence at this level for a kind of "second-order" isomorphism between internal representations and their corresponding external objects.

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(Accepted May 12, 1969)